STUDY MATERIAL

SUBJECT: ELECTRICAL POWER TRANSMISSION & DISTRIBUTION

(MODULE-2)

SEMESTER:5TH

BRANCH: EE / EEE

CONTENTS:

- > SHORT TYPE QUESTIONS AND ANSWERS
- > LONG TYPE QUESTIONS AND ANSWERS

DEPARTMENT OF ELECTRICAL ENGINEERING SRINIX COLLEGE OF ENGINEERING, BALASORE

(www.srinix.org)

Electrical power transmission and distribution

Module- 2

1. WHAT ARE THE TYPES CONDUCTORS USED IN OVERHEAD LINES?

Ans: normally we use aluminum conductor in O.H line .the aluminum conductors are of

- (a) AAC-all aluminum conductor
- (b) AAAC-all aluminum alloy conductor
- (c) ACSR-aluminum conductor steel reinforced

2. WHAT IS THE ADVANTAGE THAT ALUMINIUM IS CONSIDERED OVER COPPER?

Ans: the reason is that

A-low weight

B-low conductivity

C-low cost

3. WHY ACSR CONDUCTOR ARE PREFERRED IN TRANSMISSION LINE?

- (a) Mechanical strength is high, the length of the span can be increased and cost of erection and maintenance can be reduced.
- (b) Corona loss and skin effect are reduced.
- (c) Steel core aluminum conductor are normally cheaper than copper conductor of equal resistance

4. WHAT DO YOU MEAN BY INDUCTANCE OF A CONDUCTOR?

Ans: the inductance is defined as the flux linkage per unit current in a conductor.

$$L = \frac{total\ magnetic\ flux\ linkage}{current} = \frac{\lambda}{I}$$

5. IS INTERNAL FLUX IS DEPENDENT WITH THE CONDUCTOR SIZE? GIVE THE REASON WITH MATHEMATICAL EXPRESSION

Ans: internal flux is not depending upon the conductor size and it is due to the reason that

$$\lambda_{int} = \frac{\mu_0}{8\pi} I$$

Where I is the current in the conductor

6. WHAT DO YOU MEAN BY SKIN EFFECT?

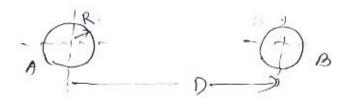
Ans: the tendency of alternating current to concentrate near the surface of a conductor is known as skin effect .due to this the crossectional area of the conductor through which current flow is reduced, the resistance of the conductor is slightly increased when carrying current .in a conductor carrying direct current there is no skin effect.

7. WHAT DO YOU MEAN BY PROXIMITY EFFECT?

Ans: the alternating magnetic flux in a conductor caused by the current flowing in a neighboring conductor which cause an apparent increase of resistance of a conductor.

8. TWO CONDUCTORTS ARE SEPARATED BY A DISTANCE OF D AND EACH OF RADIUS I .WHAT WILL BE EXTERNAL FLUX LINKAGEIF ONE CONDUCTOR ACTS AS RETURN PATH IF R<<D WHAT IS THE FLUX LINKAGE?

Ans: fig:



 λ_{EXt} =external flux linkage

$$\frac{\mu_0 I}{2\lambda} \ln \frac{D-R}{R} Wb - T/m$$

If R << D then D-R~ D

$$\lambda_{\text{ext}} = \frac{\mu_0 I}{2\lambda} \ln \frac{D}{R}$$

9. WHAT DO YOU MEAN BY G.M.R AND WHAT IS ITS EXPRESSION?

ANS:

$$L_{1} = 2 \times 10^{-7} \left(\frac{1}{4} + \ln \frac{D}{R}\right)$$

$$= 2 \times 10^{-7} \left(\ln e^{\frac{1}{4}} + \ln \frac{D}{R}\right)$$

$$= 2 \times 10^{-7} \ln \frac{D}{e^{\frac{1}{4}}R} = 2 \times 10^{-7} \ln \frac{D}{R}$$

Where r' =G,M,R i.e. geometrical mean radius

$$=e^{\frac{1}{4}}R = 0.7788R$$

R' is the value which is 0.7788 times radius in order to account for internal flux linkage.

10. WHAT IS BUNDLED CONDUCTOR AND WHAT IS THE ADVANTAGE?

ANS: bundled conductor is a conductor made up of two or more sub-conductor and is used as one phase conductor.

Advantage:

- (a) reduced surge impedance
- (b) reduced reactance
- (c) reduced voltage gradient
- (d) reduced corona loss
- (e) reduced radio interference

11. WHAT IS COMPOSITE CONDUCTOR?

ANS: The standard conductors which are normally used for transmission line operating at high voltage are known as composite conductor. They are so called as they consist of two or more element or strands electrically in parallel.

12. WHAT ARE FACTOR EFFECT SKIN EFFECT AND PROXIMITY EFFECT?

ANS: both skin and proximity effect depend upon the following factor

- (a) Size of conductor
- (b) Distance between the conductors
- (c) Frequency of the current
- (d) Permeability of the conductor material.

13. WHAT DO YOU MEAN BY TRANSPOSITION OF CONDUCTOR?

Ans: when conductors are unsymmetrical spaced under such condition the flux linkage and inductance of each phase are not the same and subsequently voltage drops in three phases are unequal even if the currents in the conductors are balanced. In order to make voltages drops equal in all conductor, we generally interchange the position of the conductors at regular interval along the lines so that each conductor occupies the original position of every other conductor over an equal distance. Such change of conductor position is called transposition.

14.CALCULATE THE CAPACITANCE OF A 100 KM LONG 3-PHASE,50 HZ OVERHEAD LINCONSISTING 3 CONDUCTOR, EACH OF DIAMETER 2CM AND SPACED 2.5 M AT THE CORNER OF AN EQUILATERAL TRIANGLE.

Ans: equilateral spacing, d=2.5m=250 cm

Radius of conductor, r=2/2=1 cm

Capacitance of each conductor to neutral

=0.0096 μF/Km

ans.

15. HOW ELECTRICAL POTENTIAL IS RELETED WITH CAPACITANCE EXPLAIN?

Ans: due to electrical potential produced between two conductor in a transmission line the capacitance

C=O/V

Where V=potential

Q=charge accumulated in the insulating medium.

16. WHAT IS PER UNIT SYSTEM &WHAT IS THE ADVANTAGE OF PER UNIT SYSTEM?

Definition: The per unit value of any quantity is defined as the ratio of actual value in any unit and the base or reference value in the same unit. Any quantity is converted into per unit quantity by dividing the numeral value by the chosen base value of the same dimension. The per unit value are dimensionless.

 $Per Unit Value = \frac{Actual value in any unit}{Base or reference value in the same unit}$

There are mainly two advantages of using the Per Unit System.

- The parameter of the rotating electrical machines and the transformer lies roughly in the same range of numerical values irrespective of their ratings if expressed in per unit system of their ratings.
- It relieves the analyst of the need to refer circuit quantities to one or other side of the transformer, making the calculations easy.

Taking the example of a transformer having the resistance in the per unit as R_{pu} ohm and the reactance as X_{pu} in ohm with referred to primary then per unit values will be

Long type[6 mark each]

1. DERIVE THE EXPRESSION OF INDUCTANCE OF SINGLE PHASE TRANSMISSION LINE.

Ans:

Inductance of Single Phase Two Wire System consists of two parallel conductors which form a rectangular loop of one turn. When an alternating current flows through such a loop, a changing magnetic flux is set up. The changing flux links the loop and hence the loop (or single phase line) possesses inductance. It may appear that inductance of a single phase line is negligible because it consists of a loop of one turn and the flux path is through air of high reluctance. But as the X-sectional area of the loop is very large, even for a small flux density, the total flux linking the loop is quite large and hence the line has appreciable Inductance of Single Phase Two Wire System.

Consider a single phase overhead line consisting of two parallel conductors A and B spaced d meters apart as shown in Fig. Conductors A and B carry the same amount of current (i.e. $I_A = I_B$), but in the opposite direction because one forms the return circuit of the other i.e. $I_A + I_B = 0$



Fig. 9.7

In order to find the inductance of conductor A (or conductor B), we shall have to consider the flux linkages with it. There will be flux linkages with conductor A due to its own current I_A and also due to the mutual inductance effect of current I_B in the conductor B.

Flux linkages with conductor A due to its own current

$$= \frac{\mu_0 I_A}{2\pi} \left(\frac{1}{4} + \int_r^\infty \frac{dx}{x} \right) \dots (i)$$

Flux linkages with conductor A due to current IB

$$= \frac{\mu_0}{2\pi} \int_d^\infty \frac{dx}{x} \qquad ...(ii)$$

Total flux linkages with conductor A is

$$\begin{split} \Psi_A &= \exp. (i) + \exp (ii) \\ &= \frac{\mu_0}{2\pi} \frac{I_A}{\left(\frac{1}{4} + \int_r^{\infty} \frac{dx}{x}\right) + \frac{\mu_0}{2\pi} \int_d^{\infty} \frac{dx}{x}}{x} \\ &= \frac{\mu_0}{2\pi} \left[\left(\frac{1}{4} + \int_r^{\infty} \frac{dx}{x}\right) I_A + I_B \int_d^{\infty} \frac{dx}{x} \right] \\ &= \frac{\mu_0}{2\pi} \left[\left(\frac{1}{4} + \log_e \infty - \log_e r\right) I_A + \left(\log_e \infty - \log_e d\right) I_B \right] \\ &= \frac{\mu_0}{2\pi} \left[\left(\frac{I_A}{4} + \log_e \infty \left(I_A + I_B\right) - I_A \log_e r - I_B \log_e d\right) \right] \\ &= \frac{\mu_0}{2\pi} \left[\frac{I_A}{4} - I_A \log_e r - I_B \log_e d \right] \quad (\because I_A + I_B = 0) \\ &I_A + I_B = 0 \quad \text{or} \quad -I_B = I_A \\ &-I_B \log_e d = I_A \log_e d \end{split}$$

7 6

Inductance of conductor A,

$$L_{A} = \frac{\Psi_{A}}{I_{A}}$$

$$= \frac{\mu_{0}}{2\pi} \left[\frac{1}{4} + \log_{e} \frac{d}{r} \right] H/m = \frac{4\pi \times 10^{-7}}{2\pi} \left[\frac{1}{4} + \log_{e} \frac{d}{r} \right] H/m$$

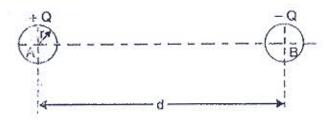
$$L_{A} = 10^{-7} \left[\frac{1}{2} + 2\log_{e} \frac{d}{r} \right] H/m \qquad ...(i)$$
Loop inductance = $2L_{A}H/m = 10^{-7} \left[1 + 4\log_{e} \frac{d}{r} \right] H/m$

$$Loop inductance = 10^{-7} \left[1 + 4\log_{e} \frac{d}{r} \right] H/m \qquad ...(ii)$$

Note that eqⁿ (ii) is the Inductance of Single Phase Two Wire System and is sometimes called loop inductance. However, inductance given by eqⁿ(i) is the inductance per conductor and is equal to half the loop inductance.

2. Derive the expression of capacitance of single phase transmission line.

Consider a Capacitance of a Single Phase Two Wire Line consisting of two parallel conductors A and B spaced d meters apart in air. Suppose that radius of each conductor is r meters is shown in Fig. below Let their respective charge be + Q and – Q coulombs per meter length.



The total potential difference between conductor A and neutral "infinite" plane is

$$V_A = \int_{r}^{\infty} \frac{Q}{2\pi x \varepsilon_0} dx + \int_{d}^{\infty} \frac{-Q}{2\pi x \varepsilon_0} dx$$
$$= \frac{Q}{2\pi \varepsilon_0} \left[\log_e \frac{\infty}{r} - \log_e \frac{\omega}{d} \right] \text{volts} = \frac{Q}{2\pi \varepsilon_0} \log_e \frac{d}{r} \text{ volts}$$

Similarly potential difference between conductor B and neutral "infinite" plane is

$$V_{B} = \int_{r}^{\infty} \frac{-Q}{2\pi x \, \varepsilon_{0}} \, dx + \int_{d}^{\infty} \frac{Q}{2\pi x \, \varepsilon_{0}} \, dx$$
$$= \frac{-Q}{2\pi \, \varepsilon_{0}} \left[\log_{e} \frac{\infty}{r} - \log_{e} \frac{\infty}{d} \right] = \frac{-Q}{2\pi \, \varepsilon_{0}} \log_{e} \frac{d}{r} \text{ volts}$$

Both these potentials are w. r. t. the same neutral plane. Since the unlike charges attract each other, the potential difference between the conductors is

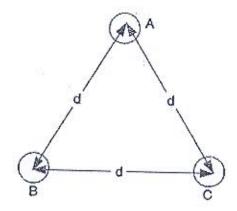
$$V_{AB} = 2V_A = \frac{2Q}{2\pi \,\varepsilon_0} \log_e \frac{d}{r} \text{ volts}$$

$$C_{AB} = Q/V_{AB} = \frac{Q}{\frac{2Q}{2\pi \,\varepsilon_0} \log_e \frac{d}{r}} \text{ F/m}$$

$$C_{AB} = \frac{\pi \,\varepsilon_0}{\log_e \frac{d}{r}} \text{ F/m} \qquad ...(i)$$

3. Find the expression of capacitance of a three phase transmission line having symmetrical spacing.

Ans: fig



The figure shows the three conductors A, B and C of the 3-phase overhead transmission line having charges Q_A , Q_B and Q_C per meter length respectively. Let the conductors be equidistant d meters from each other. We shall find the capacitance from line conductor to neutral in this symmetrically spaced line. Referring to Fig. the overall potential difference between conductor A and infinite neutral plane is given by

$$\begin{aligned} V_A &= \int_r^\infty \frac{Q_A}{2 \pi x \varepsilon_0} \, dx + \int_d^\infty \frac{Q_B}{2 \pi x \varepsilon_0} \, dx + \int_d^\infty \frac{Q_C}{2 \pi x \varepsilon_0} \, dx \\ &= \frac{1}{2 \pi \varepsilon_0} \left[Q_A \log_e \frac{1}{r} + Q_B \log_e \frac{1}{d} + Q_C \log_e \frac{1}{d} \right] \\ &= \frac{1}{2 \pi \varepsilon_0} \left[Q_A \log_e \frac{1}{r} + (Q_B + Q_C) \log_e \frac{1}{d} \right] \end{aligned}$$

Assuming balanced supply, we have

$$\begin{aligned} Q_A + Q_B + Q_C &= 0 \\ Q_B + Q_C &= -Q_A \end{aligned}$$

$$V_A = \frac{1}{2\pi\epsilon_0} \left[Q_A \log_e \frac{1}{r} - Q_A \log_e \frac{1}{d} \right] = \frac{Q_A}{2\pi\epsilon_0} \log_e \frac{d}{r} \text{ volts}$$

Capacitance of conductor "A" w.r.t neutral

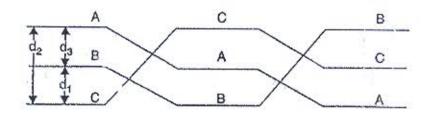
$$C_A = \frac{Q_A}{V_A} = \frac{Q_A}{\frac{Q_A}{2\pi \, \epsilon_0} \log_e \frac{d}{r}} \, \text{F/m} = \frac{2\pi \, \epsilon_0}{\log_e \frac{d}{r}} \, \text{F/m}$$

$$C_A = \frac{2\pi \, \epsilon_0}{\log_e \frac{d}{r}} \, \text{F/m}$$

4. Find the expression of capacitance of a three phase transmission line having unsymmetrical spacing

Ans:

The figure shows a 3-phase transposed line having unsymmetrical spacing. Let us assume balanced conditions i.e. $Q_A + Q_B + Q_C = 0$.



Considering all the three sections of the transposed line for phase A

Potential of 1st position,

$$V_1 = \frac{1}{2\pi\epsilon_0} \left(Q_A \log_e \frac{1}{r} + Q_B \log_e \frac{1}{d_3} + Q_C \log_e \frac{1}{d_2} \right)$$

Potential of 2nd position,

$$V_2 = \frac{1}{2\pi\epsilon_0} \left(Q_A \log_e \frac{1}{r} + Q_B \log_e \frac{1}{d_1} + Q_C \log_e \frac{1}{d_3} \right)$$

Potential of 3rd position,

$$V_3 = \frac{1}{2\pi\epsilon_0} \left(Q_A \log_e \frac{1}{r} + Q_B \log_e \frac{1}{d_2} + Q_C \log_e \frac{1}{d_1} \right)$$

Average voltage on conductor A is

As
$$Q_A + Q_B + Q_C = 0$$
, therefore, $Q_B + Q_C = -Q_A$

$$\therefore V_A = \frac{1}{6\pi\epsilon_0} \left[Q_A \log_e \frac{1}{r^3} - Q_A \log_e \frac{1}{d_1 d_2 d_3} \right]$$

$$= \frac{Q_A}{6\pi\epsilon_0} \log_e \frac{d_1 d_2 d_3}{r^3}$$

$$= \frac{1}{3} \times \frac{Q_A}{2\pi\epsilon_0} \log_e \frac{d_1 d_2 d_3}{r^3}$$

$$= \frac{Q_A}{2\pi\epsilon_0} \log_e \left(\frac{d_1 d_2 d_3}{r^3} \right)^{1/3}$$

$$= \frac{Q_A}{2\pi\epsilon_0} \log_e \frac{(d_1 d_2 d_3)^{1/3}}{r}$$

Capacitance from conductor to neutral is

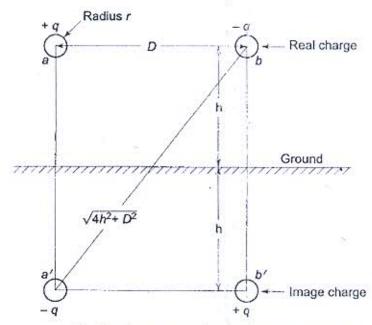
$$C_A = \frac{Q_A}{V_A} = \frac{2 \pi \varepsilon_C}{\log_c \frac{\sqrt[3]{d_1 d_2 d_3}}{r}} F/m$$

5. Discuss the Effect of Earth on Transmission Line Capacitance.

In calculating the Effect of Earth on Transmission Line Capacitance, the presence of earth was ignored, so far. The effect of earth on capacitance can be conveniently taken into account by the method of images.

Method of Images

The electric field of transmission line conductors must conform to the presence of the earth below. The earth for this purpose may be assumed to be a perfectly conducting horizontal sheet of infinite extent which therefore acts like an equipotential surface.



Single-phase transmission line with images

Consider a single-phase line shown in Fig. It is required to calculate its capacitance taking the presence of earth into account by the method of images described above. The equation for the voltage drop V_{ab} as determined by the two charged conductors a and b, and their images a' and b' can be written as follows:

$$\begin{split} V_{ab} &= \frac{1}{2 \, \pi k} \bigg[\, q_a \, \ln \frac{D}{r} + q_b \, \ln \frac{r}{D} + q_{a'} \, \ln \frac{(4h^2 + D^2)^{1/2}}{2h} \\ &\quad + q_{b'} \, \ln \frac{\dot{2}h}{(4h^2 + D^2)^{1/2}} \bigg] \end{split}$$

Substituting the values of different charges and simplifying, we get

$$V_{ab} = \frac{q}{\pi k} \ln \frac{2hD}{r(4h^2 + D^2)^{1/2}}$$

It immediately follows that

$$C_{ab} = \frac{\pi k}{\ln \frac{D}{r(1 + (D^2/4h^2))^{1/2}}}$$
 F/m line-to-line

And

$$C_n = \frac{2 \pi k}{\ln \frac{D}{r(1 + (D^2/4h^2))^{1/2}}}$$
 F/m to neutral

It is observed from the above equation that the presence of earth modifies the radius r to $r(1 + (D^2/4h^2))^{1/2}$. For h large compared to D (this is the case normally), the effect of earth on line capacitance is of negligible order.

6. Derive the expression of inductance of a three phase unsymmetrical transmission line. Assume the line is transposed.

When 3-phase line conductors are not equidistant from each other, the conductor spacing is said to be unsymmetrical. Under such conditions, the flux linkages and Inductance of 3 Phase Overhead Line of each, phase are not the same. A different inductance in each phase results in unequal voltage drops in the three phases even if the currents in the conductors are balanced. Therefore, the voltage at the receiving end will not be the same for all phases. In order that voltage drops are equal in all conductors, we generally interchange the positions of the conductors at regular intervals along the line so that each conductor occupies the original position of every other conductor over an equal distance. Such an exchange of positions is known as transposition. The phase conductors are designated as A, B and C and the positions occupied are numbered 1, 2 and 3. The effect of transposition is that each conductor has the same average inductance.

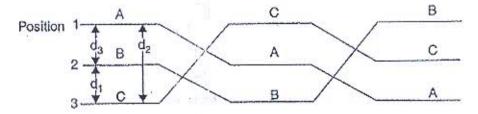


Fig shows a 3-phase transposed line having unsymmetrical spacing. Let us assume that each of the three sections is 1 m in length. Let us further assume balanced conditions i.e., $I_A + I_B + I_C = 0$. Let the line currents are:

$$I_A = I(1+j0)$$

 $I_B = I(-0.5-j0.866)$
 $I_C = I(-0.5+j0.866)$

As proved above, the total flux linkages per meter length of conductor A is

$$\Psi_{A} = \frac{\mu_{0}}{2\pi} \left[\left(\frac{1}{4} - \log_{e} r \right) I_{A} - I_{B} \log_{e} d_{3} - I_{C} \log_{e} d_{2} \right]$$

Putting the values of IA, IB and IC, we get,

$$\begin{split} \Psi_{A} &= \frac{\mu_{0}}{2\pi} \left[\left(\frac{1}{4} - \log_{e} r \right) I - I(-0.5 - j \cdot 0.866) \log_{e} d_{3} - I(-0.5 + j \cdot 0.866) \log_{e} d_{2} \right] \\ &= \frac{\mu_{0}}{2\pi} \left[\frac{1}{4} I - I \log_{e} r + 0.5 I \log_{e} d_{3} + j \cdot 0.866 \log_{e} d_{3} + 0.5 I \log_{e} d_{2} - j \cdot 0.866 I \log_{e} d_{2} \right] \\ &= \frac{\mu_{0}}{2\pi} \left[\frac{1}{4} I - I \log_{e} r + 0.5 I \left(\log_{e} d_{3} + \log_{e} d_{2} \right) + j \cdot 0.866 I \left(\log_{e} d_{3} - \log_{e} d_{2} \right) \right] \\ &= \frac{\mu_{0}}{2\pi} \left[\frac{1}{4} I - I \log_{e} r + I \cdot \log_{e} \sqrt{d_{2}d_{3}} + j \cdot 0.866 I \log_{e} \frac{d_{3}}{d_{2}} \right] \\ &= \frac{\mu_{0}}{2\pi} \left[\frac{1}{4} I + I \log_{e} \frac{\sqrt{d_{2}d_{3}}}{r} + j \cdot 0.866 \log_{e} \frac{d_{3}}{d_{2}} \right] \\ &= \frac{\mu_{0}}{2\pi} \left[\frac{1}{4} I + I \log_{e} \frac{\sqrt{d_{2}d_{3}}}{r} + j \cdot 0.866 \log_{e} \frac{d_{3}}{d_{2}} \right] \end{split}$$

Inductance of conductor A is

$$L_{A} = \frac{\Psi_{A}}{I_{A}} = \frac{\Psi_{A}}{I}$$

$$= \frac{\mu_{0}}{2\pi} \left[\frac{1}{4} + \log_{e} \frac{\sqrt{d_{2} d_{3}}}{r} + j \cdot 0.866 \log_{e} \frac{d_{3}}{d_{2}} \right]$$

$$= \frac{4\pi \times 10^{-7}}{2\pi} \left[\frac{1}{4} + \log_{e} \frac{\sqrt{d_{2} d_{3}}}{r} + j \cdot 0.866 \log_{e} \frac{d_{3}}{d_{2}} \right] \text{H/m}$$

$$= 10^{-7} \left[\frac{1}{2} + 2 \log_{e} \frac{\sqrt{d_{2} d_{3}}}{r} + j \cdot 1.732 \log_{e} \frac{d_{3}}{d_{2}} \right] \text{H/m}$$

Similarly inductance of conductors B and C will be :

$$L_B = 10^{-7} \left[\frac{1}{2} + 2 \log_e \frac{\sqrt{d_3 d_1}}{r} + j \cdot 1.732 \log_e \frac{d_1}{d_3} \right] \text{H/m}$$

$$L_C = 10^{-7} \left[\frac{1}{2} + 2 \log_e \frac{\sqrt{d_1 d_2}}{r} + j \cdot 1.732 \log_e \frac{d_2}{d_1} \right] \text{H/m}$$

Inductance of each line conductor

$$= \frac{1}{3} (L_A + L_B + L_C)$$

$$= \left[\frac{1}{2} + 2\log_e \frac{\sqrt[3]{d_1 d_2 d_3}}{r} \right] \times 10^{-7} \text{ H/m}$$

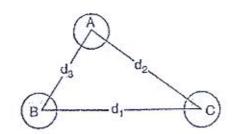
$$= \left[0.5 + 2\log_e \frac{\sqrt[3]{d_1 d_2 d_3}}{r} \right] \times 10^{-7} \text{ H/m}$$

If we compare the formula of inductance of an asymmetrically spaced transposed line with that of symmetrically spaced line, we find that Inductance of 3 Phase Overhead Line of each line conductor in the two cases will be equal if $d=\sqrt[3]{d_1d_2d_3}$. The distance d is known as equivalent equilateral spacing for unsymmetrical transposed line.

7.find the expression of inductance of three phase transmission line having symmetrical spacing.

Ans

The Fig shows the three conductors A, B and C of a 3 Phase Overhead Line carrying currents I_A , I_B and I_C respectively. Let d_1 , d_2 and d_3 be the spacing's between the conductors as shown. Let us further assume that the loads are balanced i.e. $I_A + I_B + I_C = 0$. Consider the flux linkages with conductor A. There will be flux linkages with conductor A due to its own current and also due to the mutual inductance effects of I_B and I_C .



Flux linkages with conductor A due to its own current

$$= \frac{\mu_0 I_A}{2\pi} \left(\frac{1}{4} + \int_r^{\infty} \frac{dx}{x} \right) \dots (i)$$

Flux linkages with conductor A due to current I_B

$$= \frac{\mu_0 I_B}{2\pi} \int_{d_3}^{\infty} \frac{dx}{x} \dots (ii)$$

Flux linkages with conductor A due to current $I_{\mathbb{C}}$

$$= \frac{\mu_0 I_C}{2\pi} \int_{d_2}^{\infty} \frac{dx}{x} \qquad ...(iii)$$

Total flux linkages with conductor A is

$$\begin{split} \Psi_{A} &= (i) + (ii) + (iii) \\ &= \frac{\mu_{0} I_{A}}{2\pi} \left(\frac{1}{4} + \int_{r}^{\infty} \frac{dx}{x} \right) + \frac{\mu_{0} I_{B}}{2\pi} \int_{d_{3}}^{\infty} \frac{dx}{x} + \frac{\mu_{0} I_{C}}{2\pi} \int_{d_{2}}^{\infty} \frac{dx}{x} \\ &= \frac{\mu_{0}}{2\pi} \left[\left(\frac{1}{4} + \int_{r}^{\infty} \frac{dx}{x} \right) I_{A} + I_{B} \int_{d_{3}}^{\infty} \frac{dx}{x} + I_{C} \int_{d_{2}}^{\infty} \frac{dx}{x} \right] \\ &= \frac{\mu_{0}}{2\pi} \left[\left(\frac{1}{4} - \log_{e} r \right) I_{A} - I_{B} \log_{e} d_{3} - I_{C} \log_{e} d_{2} + \log_{e} \infty \left(I_{A} + I_{B} + I_{C} \right) \right] \\ I_{A} + I_{B} + I_{C} &= 0, \end{split}$$

$$\Psi_{A} &= \frac{\mu_{0}}{2\pi} \left[\left(\frac{1}{4} - \log_{e} r \right) I_{A} - I_{B} \log_{e} d_{3} - I_{C} \log_{e} d_{2} \right]$$

For symmetrical spacing $d1 = d_2 = d_3 = d$.under such condition

$$\begin{split} \psi_A &= \frac{\mu_0}{2\pi} \bigg[\bigg(\frac{1}{4} - \log_e r \bigg) I_A - I_E \log_e d - I_C \log_e d \bigg] \\ &= \frac{\mu_0}{2\pi} \bigg[\bigg(\frac{1}{4} - \log_e r \bigg) I_A - \big(I_B + I_C \big) \log_e d \bigg] \\ &= \frac{\mu_0}{2\pi} \bigg[\bigg(\frac{1}{4} - \log_e r \bigg) I_A + I_A \log_e d \bigg] \qquad (\because I_B + I_C = -I_A) \\ &= \frac{\mu_0}{2\pi} \bigg[\frac{1}{4} + \log_e \frac{d}{r} \bigg] \text{ werber-turns/m} \end{split}$$

Inductance of conductor A,

$$L_A = \frac{\Psi_{A'}}{I_A} H / m = \frac{\mu_0}{2\pi} \left[\frac{1}{4} + \log_e \frac{d}{r} \right] H / m$$

$$= \frac{4\pi \times 10^{-7}}{2\pi} \left[\frac{1}{4} + \log_e \frac{d}{r} \right] H / m$$

$$L_A = 10^{-7} \left[0.5 + 2\log_e \frac{d}{r} \right] H / m$$

Similar way, the expressions for inductance is the same for conductors B and €.